Integrable Models in Field/String Theory

George Jorjadze
Razmadze Mathematical Institute of TSU & Free University of Tbilisi

45 hours: 15 Lectures (30 hours) and 15 hours of practical training

1. Hamilton's principle. Noether's theorem. Gauge symmetry. Noether's second theorem. Hamiltonian description. Faddeev- Jackiw formalism.
2. Symplectic geometry. Definitions and notations. Useful formulas and identities. Hamiltonian vector fields. Darboux theorem. Symplectic structure on TQ, T^*Q and on the space of solutions. Moment map.
3. The SL(2, R) group. The sl(2, R) algebra. The Killing form. The exponential map. The adjoint representation. Functions, vector fields, 1-forms and the metric on the SL(2, R) group manifold. Co- cycles of Lie algebras and central extensions.
4. Particle dynamics on symmetric spaces. The Liouville model. The dynamics of a particle in SU(2). Relativistic particle in SL(2, R). Particle dynamics in AdS space. The dynamics of a massless particle.
5. Gauging and Hamiltonian reduction. Gauging of Noether symmetries. Singular Lagrangian. First order formalism. Reductions of differential forms. Examples: Mechanical model of QED, gauging of the particle dynamics on SU(2) and SL(2, R).
6. The method of co-adjoint orbits. Co-adjoint representation of Lie groups. Co-adjoint orbits. Symplectic forms and Hamiltonian vector fields on co-adjoint orbits. Geometric quantization. Choice of polarization. Irreducible representations.
7. Geometric quantization and coherent states. Symmetries and coherent states. Examples: Weyl group, SL(2, R) and SU(2) coherent states. Symbol calculus. Moyal quantization. Coherent state formalism and geometric quantization.
8. The Lagrangian formulation of SL(2, R) WZW theory. The SL(2, R) target space. The SL(2, R) WZW Lagrangian. The general solution and global symmetries. The SU(2) WZW Lagrangian and the WZ term. Symmetries and integration of dynamical equations.
9. The Symplectic structure of 2d free-field theory. Free field theory on a cylinder and on a strip. Chiral fields and the chiral symplectic form. The Poisson brackets algebra of chiral fields. Vertex functions. The energy momentum tensor and the conformal symmetry.
10. The Hamiltonian formulation of WZW theory. Canonical structure of WZW theory. The chiral symplectic form. The Poisson brackets algebra of chiral WZ fields. Kac-Moody algebra. The Sugawara energy momentum tensor. SU(2) and SL(2, R) WZW models.
11. Gauging of WZW theory. Vector and axial gauging of SL(2, R) WZW theory. U(1) gauging and SL(2, R)/U(1) black hole model. R 1 gauging. Nilpotent gauging and Liouville theory. Hamiltonian reduction and free-field parameterization.
12. Canonical quantization of 2d CFT. Canonical quantization of free- field theory. 2d conformal symmetry and Virasoro algebra. Vertex operators and their algebra.

Canonical map to Liouville theory. Construction of Liouville vertex operators and calculation of the reflection amplitude.

13. Geometric quantization of infinite dimensional symmetries. The co- adjoint orbits of Virasoro group. Symplectic structure and Poisson brackets. Transformation to free-field variables. Coherent states of infinite dimensional translation group and 2d conformal group. Transition amplitudes between the coherent states. Kac-Moody group.
14. String dynamics in Minkowsky space Dynamical equations and gauge fixing. Integration of equations of motion. Light-cone gauge quantization. Covariant quantization and critical dimension. Polyakov method and non-critical strings. Static gauge quantization.
15. The AdS/CFT correspondence. String dynamics in AdS and AdS×S spaces. Lax pair representation of the dynamical equations. Integration by the Pohlmeyer method. Static gauge quantization of AdS strings. N = 4 supersymmetric Yang-Mills theory. Main ideas of the correspondence. Integrable structures of the dual theories.

References

[1] K. Pohlmeyer, "Integrable Hamiltonian Systems And Interactions Through Quadratic Constraints," Commun. Math. Phys. 46 (1976) 207.

[2] K. Sundermeyer, Lect. Notes Phys. 169 (1982) 1.

[3] E. Witten, "Nonabelian bosonization in two dimensions," Commun. Math. Phys. 92, 455 (1984).

[4] A. M. Perelomov, "Generalized coherent states and their applications," Berlin, Germany: Springer (1986) 320 p.

[5] M. B. Green, J. H. Schwarz and E. Witten, "Superstring Theory. Vol. 1: Introduction," Cambridge, Uk: Univ. Pr. (1987) 469 P. (Cambridge Monographs On Mathematical Physics).

[6] L. D. Faddeev and R. Jackiw, "Hamiltonian Reduction of Unconstrained and Constrained Systems," Phys. Rev. Lett. 60 (1988) 1692.

[7] N.M.J. Woodhouse, Geometric Quantisation, Clarendon, Oxford, 1992.

[8] P. Di Francesco, P. Mathieu and D. Snchal, "Conformal Field Theory, Springer-Verlag, " New York, 1997.

[9] J. M. Maldacena, "The large N limit of superconformal field theories and supergravity," Adv. Theor. Math. Phys. 2 (1998) 231 [Int. J. Theor. Phys. 38 (1999) 1113] [arXiv:hepth/9711200].

[10] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, "Large N field theories, string theory and gravity," Phys. Rept. 323 (2000) 183.

[11] J. Teschner, "Liouville theory revisited," Class. Quant. Grav. 18 (2001) R153 [arXiv:hepth/0104158].

[12] A. Rogers, "Supermanifolds: Theory and applications," Hackensack, USA: World Scientific (2007) 251 p.

[13] M. Grigoriev and A. A. Tseytlin, "On reduced models for superstrings on AdSn × S n ," Int. J. Mod. Phys. A 23 (2008) 2107.

[14] N. Beisert, C. Ahn, L. F. Alday, Z. Bajnok, J. M. Drummond, L. Freyhult, N. Gromov and

R. A. Janik et al., “Review of AdS/CFT Integrability: An Overview,” Lett. Math. Phys. 99, 3 (2012).
[15] L. V. Prokhorov and S. V. Shabanov, “Hamiltonian mechanics of gauge systems,” Cambridge, UK: Univ. Pr. (2011) 466 p.
[16]. G. Jorjadze, Lectures on Integrability in Field and String Theory (Electronic course).